CENTRIFUGE AND CENTRIFUGAL FORCE

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I. THE LAWS OF CIRCULAR MOTION

The laws of circular motion were discovered and formulated by the English mathematician and physicist Isaac Newton (1642 - 1727).

Principle of inertia

'*A moving mass on which no force acts, describes a perfectly rectilinear trajectory*.'

Force concept

'*A force refers at any cause capable of deflecting the trajectory of a mass.*'

Principle of reciprocity

'*Any mass on which a force is acting, responds by a reciprocal action of equal intensity, but of opposite direction.*'

These three laws allow to describe any circular motion:

- Movement of the Earth around the Sun.

- Movement of a centrifuge.

- Movement of a ball that turns around an axis.

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II. EARTH TURNS AROUND THE SUN

Universal laws

The laws of motion, discovered by the English physicist Isaac Newton, are universal laws: they apply in the same way to the balance of the solar system, to the centrifuge, to the ball which turns around an axis, or to any other phenomenon, that is the genius of Isaac Newton.

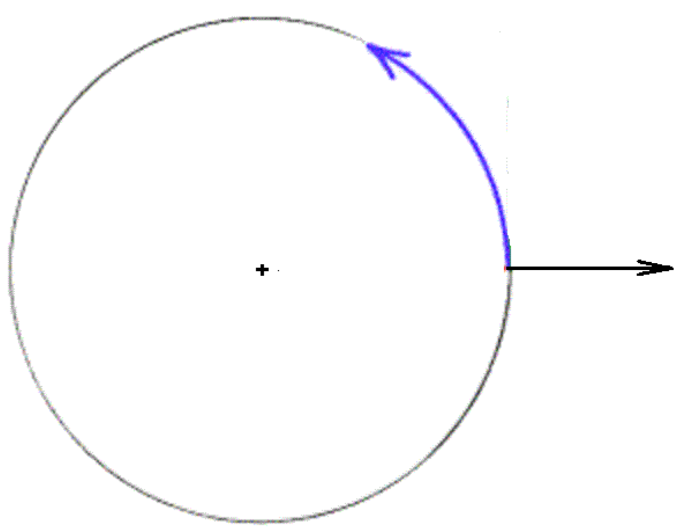
Newton's first law states that the natural trajectory of a moving body is perfectly rectilinear. If this move is deviated, it is because of a force.

The notion of force stems from the previous principle: a force is any cause acting on a mass by deflecting its trajectory. A force can act at a distance or by contact.

The centrifugal force

What is a so-called centrifugal force? Centrifugal means "*which moves away from the center*", "*which escapes from the center*".

A force qualified as centrifugal should be able to act on a mass in a radial direction, that is to say in the direction indicated by the extension of a radius.



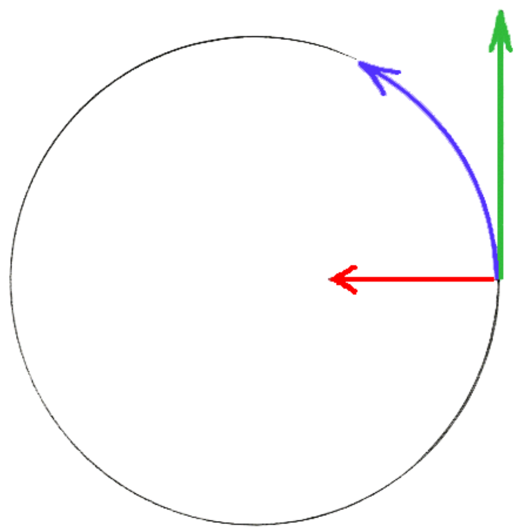
Centrifugal force

When a mass describes a circular path (blue arrow), the centrifugal force should act in the direction indicated by the extension of a radius (black arrow).
Such a move has never been observed.

The movement of the Earth

Let us apply this law to the movement of the Earth which orbits the Sun at a speed of 30 kilometers per second on a circular path radius of 150 million kilometers.

How to explain this trajectory? It is due to the gravitational force created by the mass of the Sun. This mysterious force (also called centripetal force) acts at a distance, it is of the same nature as that which, on Earth, makes the masses fall on the ground.



Schematic representation of the motion of the Earth.

Earth describes a circular path (blue arrow) because of the gravitational force (red arrow) which attracts towards the sun. If this force did not exist, the Earth would take a straight path (green arrow). There is a centripetal force but no centrifugal one. Be careful not to add force, speed and trajectory!

Change the laws of the Universe ...

Imagine that the laws of the universe are completely changed. What would happen if the Earth had a zero speed or suddenly ceased to obey the attraction of the Sun?

- without speed but driven by gravitation, the Earth immediately would take the direction of the Sun, until to dive into it.

- without gravitation but retaining its speed, the Earth immediately would adopt a straight path and move away from the Sun.

Insist on this point: if the Earth was moving away from the Sun, it would be a gravitational force default and not because of any centrifugal force.

The third principle of Isaac Newton

Newton's principle states that any force acting on a mass causes a reciprocal action of equal intensity but opposite direction.

Does this principle apply in the case of the solar system? Yes of course!

The Sun attracts the Earth, so the Earth attracts the Sun, with the same intensity but opposite! This force acts at the center of the Sun, it is oriented towards to the center of the Earth.

Why the Earth deflects its trajectory, the Sun remaining perfectly still? It is a question of strength between two unequal masses: the Sun has 333,333 times more mass than the Earth[(*)].

Conclusion

The concept of centrifugal force is no more necessary in this description than in any other!

(*) *Mass of the Sun (S): 2×10^{30} kg; mass of the Earth (T): 6×10^{24} kg; ratio S / T: 0.33×10^{6}.*

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III. THE CENTRIFUGE

Everyone knows the principle of the centrifuge which is used in a washing machine or washer dryer.

Centrifuge comes from ‘centrifugal’ that just means 'which moves away from the center.'

Does the centrifuge prove the existence of centrifugal force?

This is what we'll see ...

A fundamental principle ...

A fundamental principle of physics (this is the Newton's first law) provides that the normal trajectory of a moving body is perfectly rectilinear.

This natural path can only be deflected by a transverse force. The concept of force follows from the previous principle: a force is any cause able to deflect a trajectory.

There are two types of forces corresponding to this definition: the gravitational force acting at a distance and the guiding force that acts by contact.

These laws are the foundation of the modern physics, they were discovered by the English physicist Isaac Newton in 1666.

How does a centrifuge?

The essential part of a centrifuge is a drum, a kind of hollow cylinder with holes on surface to let the water out, and an electric motor. The electric motor drives the drum in a circular motion.

The circular motion of the drum forces the wet laundry to describe a circular path.

According to Newton's law, the laundry in motion should take a straight path. If the laundry describes a circular path, it is because it is constantly driven by the force of the drum. This force acts by contact and neither at a distance nor on the center of gravity (see ADILCA folder '*centripetal force*').

Thus, no centrifugal or centripetal force therefore acts on the laundry inside the drum, at any time whatsoever.

The drainage

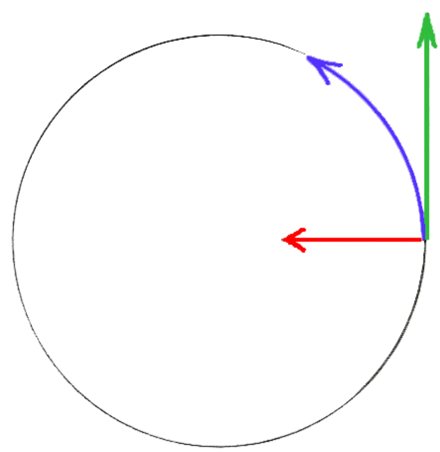
How the water comes out of the drum?

It has been said that the drum is a cylinder with holes. The machine can communicate a force only to water molecules in contact with the drum. When water molecules reach the holes, the contact force is zero, so they immediately adopt a straight path that leads them outside of the drum.

This trajectory is not radial, that is to say in the extension of the radius, but tangential, i.e. perpendicular to the radius of the drum. To be precise, the trajectory of the water molecules is perfectly straight as when they leave the drum, but gradually this path is influenced by the combined action of the gravity and the air resistance.

Thus, if the water comes out of the drum, it is because of no force and not because of the centrifugal force.

Obviously, no centrifugal force affects the water molecules at any time whatsoever, neither inside the drum, nor outside the drum.



Principle of the centrifuge

Wet laundry in a circle (blue arrow) through the contact force (red arrow) exerted by the drum. Water leaves the drum on a straight line tangent to the radius (green arrow). There is neither centripetal force nor centrifugal force. Be careful not to add force, speed and trajectory!

Conclusion

The centrifuge exists, but not the centrifugal force!

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IV. A BALL THAT TURNS AROUND AN AXIS

Some 'pseudo-physicists'[(1)], probably anxious to come to the aid of the concept of centrifugal force, thought they could prove the existence of an imaginary force by describing the circular motion of a ball around a rotation axis! Wasted effort!

Abuse of the concept of centrifugal force comes under the manipulation or incompetence. Indeed, the centrifugal force being an imaginary force, we can and we must do without, whatever the circular motion considered: centrifuge, ball held by a string, car, earth, etc.

If we cannot, it's because there is a 'bug' somewhere! For you to find it!

Description of the device

Imagine a device such as a motor driving a vertical shaft with, at the end of the shaft, a string holding a ball. At rest, a single force acts on the ball, it is its weight. The ball remains motionless because its weight is balanced by the string. The system is in equilibrium.

The rotary system

Let start the engine. Once the system begins to rotate, the equilibrium condition is no longer respected. After the acceleration phase[(2)], as soon as the rotation speed is stabilized, the ball has moved away from the axis, it describes a circular path in a horizontal plane, the string forms an angle with respect to the vertical.

The Contenders

According to the fundamental principle of physics (Newton's first law), the ball in motion should describe a straight path. If it does not, it is because the ball is constantly driven by a force which makes this circular path.

This force is directed toward the axis[(3)], but where does it come from ?

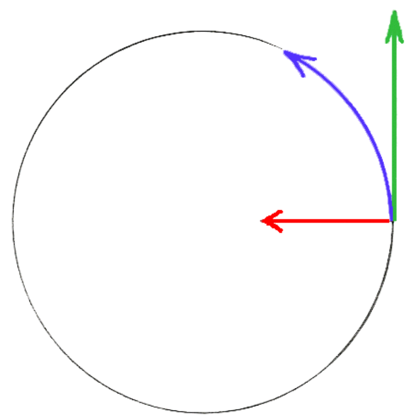
Unlike the situation observed at rest, the equilibrium now is broken because there are two forces involved in the system by the tension of the string:

- a vertical one, of equal intensity but opposite to the weight, that keeps the ball in the air;

- a horizontal one that maintains the ball on a circular path.

The string tension is equal to the resultant (in other words, to the vector sum) of these two forces.

This description is complete like that! There is no need to add anything, especially not centrifugal force. There is no need, and for a good reason: the centrifugal force does not exist!



Device seen from above: the ball describes a circular path (blue arrow) because of the horizontal force (red arrow) transmitted by the string (axis and string do not appear in the drawing). If this force suddenly stops acting, for example in the event of a string breakage, the ball escapes by a rectilinear way (green arrow). There is neither centripetal force nor centrifugal force. Never add force, speed and trajectory!

Calculation of the horizontal force

What relationship should be used to calculate the intensity of this horizontal force? Only this one:

$$\mathbf{F = M\ V^2 / R}$$

According to the International System of Units (**SI**) and to ensure the validity of the relationship, the mass M of the ball shall be expressed in kilograms (kg symbol), its speed V in meters per second ($m.s^{-1}$ symbol) and the radius R of its trajectory in meters (m symbol).

The result is expressed in newtons (symbol N), the international unit of force.

The consistency of the units:

$$\mathbf{F} = kg\ .\ (m.s^{-1})^2\ .\ m^{-1} = kg\ .\ m^2.s^{-2}\ .\ m^{-1} = kg\ .\ m.s^{-2} = \mathbf{N}$$

The third principle of Newton

The third Newton's principle states that any force acting on a mass causes a reciprocal action of equal intensity but opposite direction.

How this principle, often misunderstood, does apply in the case of the device described above? Of a very simple and very logical way: the ball in motion exerts a pull on the string, and therefore on the axis. The intensity of this traction is equal to the vector sum of the weight of the ball and the horizontal force[(4)].

If the ball goes on the circular path while the axis remains insensitive to this traction, it is either because the motor forms a substantially heavier mass than the ball, either because the device is firmly fixed to the Earth[(5)].

In fact, if it were not the case, the axis and engine would be overthrown by the motion of the ball.

A 'static' description

Imagine now the same device, this time with an observer placed on the axis and turning at the same time as him. The observer then having no longer any perception of rotation, it would require him to reason with statics, that is, to reason as if the ball stopped turning.

Twisted reasoning? Not at all! Humans have reasoned like this for millennia, until the advent of Copernic, Galilei and Newton! Indeed, the truth at that time was to describe the movement of the Sun and the planets of the solar system assuming that the Earth was motionless.

The funny thing is that this way of thinking continues today! With qualifiers that few people understand: they say that they reason in a relative, non-inertial or non-Galilean framework. In short, it is enough to say that we reason in statics, it is strictly the same reasoning!

As for the manual, do not look elsewhere! Our website is almost the only one to carefully distinguish descriptions, and taking great care never to mix them!

This reasoning is called 'static' as opposed to reasoning 'dynamic' explained above. What are the limits of this reasoning?

The ball movement is now frozen, as if watching from a photograph. How then to explain the angle of the string relative to the vertical? This is where the centrifugal force appears. Centrifugal the misnamed!

Indeed, to keep the ball in balance and explain the angle of the string relative to the vertical, imagine an imaginary force acting horizontally on the center of mass of the ball, but in a direction opposite to that of the axis! That is the centrifugal force!

The misnamed force because there is neither circular path nor center, the ball motion having disappeared in the eyes of the observer!

Let us add here, the third principle of Newton is strictly inapplicable since there is no interaction, the centrifugal force being an imaginary force.

Calculation of the centrifugal force

What relationship should be used to calculate the intensity of centrifugal force? This one, and the only one:

$$\mathbf{F' = -M\,g\ tangent\ \alpha}$$

The sign [–] is to specify the spatial orientation of the force. According to the International Units System (SI) and to ensure the validity of the relationship, the mass M of the ball shall be expressed in kilograms (kg symbol), the gravitational acceleration **g** in meters per second squared (symbol $m.s^{-2}$) and the angle α of the string with respect to the vertical, in either degrees or radians (trigonometric values have no dimension).

The result is expressed in newtons (N symbol) which is the international unit of force.

The consistency of units and checks:

$$\mathbf{F'} = kg\ .\ m.s^{-2} = \mathbf{N}$$

Equality = danger!

A numerical application of these relations give a surprising result: all other conditions being equal, we find that the horizontal force and the centrifugal force have exactly the same intensity!

However, nothing should be concluded because these two forces do not belong to the same description! The worst mistake is to confuse or mix them:

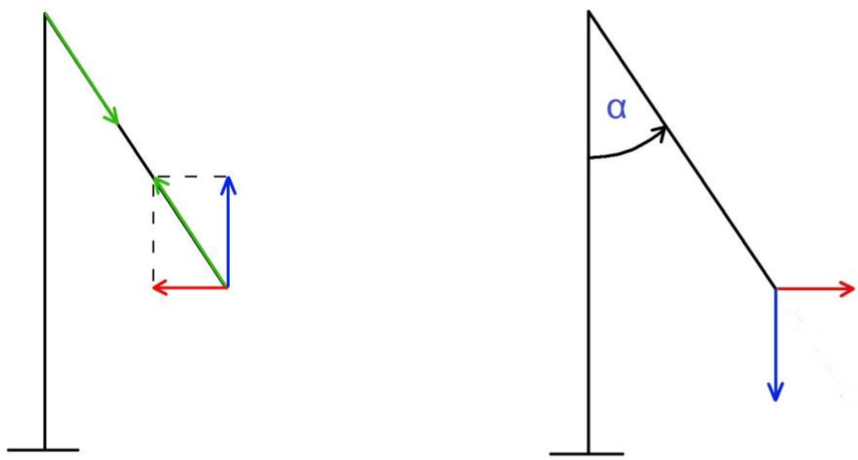
- one of these two forces is acting by contact of the string on a ball in motion, the other is acting miraculously on the center of mass of a ball motionless;

- one of these two forces is the result of an interaction, from the axis on the string and from the string on the ball, the other not;

- one of these two forces is directed towards the axis, the other is in opposite direction. To avoid any confusion, a good precaution is to fit the centrifugal force a negative sign to specify its direction which is contrary to the logic of the movement.

Two drawings to understand ...

The previous two descriptions are not to be confused or mixed, it is imperative to distinguish them by two separate drawings, like these:



First drawing: this is the dynamic description of the rotation of a ball around an axis, in a horizontal view.

- the blue arrow shows the lift force, the one that keeps the ball in vertical balance;

- the red arrow shows the horizontal force, the one that keeps the ball on a circular path;

- two green arrows: one shows the resultant of the two aforementioned forces, it comes from the axis which acts on the ball by the string; the other is the reciprocal action that is exerted on the axis by the string.

Second drawing: this is the static description, the axis does not rotate at all, the ball is stationary in space as by magic but the string shows an angle relative to the vertical.

- the blue arrow shows the weight of the ball;

- the red arrow shows the centrifugal force;

- caution: no reciprocal action here, nor addition of vectors between a real force (the weight of the ball) and an imaginary force (the centrifugal force).

Conclusion

Nothing mysterious or magical in the movement of a ball that turns around an axis.

The axis is driven by an electric motor, it is the electromagnetic force at work, as for the drum of the centrifuge.

Then, everything is explained by the contact force that the axis exerts on the ball through the wire:

- At rest, the wire is stretched vertically to balance the gravitational force that the terrestrial globe exerts on the ball.

- Once the axis starts to turn, the ball describes a circular trajectory because of the wire which prevents it from escaping in a straight line.

There is neither centripetal force nor centrifugal force.

(1) *'Pseudo-physicists': so we called, derisively, the physicists who persist in considering only the fictitious forces or pseudo-forces, ignoring the others.*

(2) *During the acceleration phase, the engine work provides a kinetic energy corresponding to the speed acquired by the ball (disregarding the mass of the string and the air resistance), to which must be added the gravitational energy corresponding to the height difference between the position of the ball at rest and its position after the stabilized speed.*

(3) *This force is not a centripetal one: indeed, the string never approaches the ball from the axis, it simply deviates it from a straight line to give a circular path (see ADILCA folder 'centripetal force').*

(4) *Disregarding the mass of the string. However, in the case of a fairground carousel, it would obviously take into account the mass of the support arm.*

(5) *This means that, in the final analysis, the real forces are all the result of an interaction with the Earth.*

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V. GENERAL CONCLUSION

The centripetal force is a real force which acts at a distance between two centers of gravity. This concept explains the rotational movement of the planets, such as that of the Earth around the Sun.

All the other rotational movements are explained by the action of a contact force, such as that exerted by the drum in the case of linen spinning in a centrifuge, or that exerted by the wire in the case of a ball rotating around an axis.

Centrifugal force is an imaginary force that has no place in any description of any circular motion as gravitation, spinning of a centrifuge or a ball spinning around an axis.

The centrifugal force is an imaginary force, provided that the real movement is frozen. The concept explains the apparent equilibrium of a system considered to be immobile.

Be careful to clearly distinguish the descriptions, never to confuse the real and the imaginary, never to mix dynamic and static descriptions, never to associate a real force with a fictitious force.

More generally, beware of formulas with uncertain instructions and calculations with identical results, they do not allow the interchangeability of concepts and reasoning.

For example, the famous magic formula $\mathbf{F = MV^2/R}$ is only valid in dynamics, never in statics, so it can only express a real force and not an imaginary force.

Finally, do not hesitate to question the certainties, demonstrations or drawings of your teachers, however friendly and competent they may be. Remember that the best of them could (unintentionally) mislead you.

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VI. CALCULATION: EARTH TURNS AROUND THE SUN

A. Dynamic description

The earth is moving, she turns around the sun.

1. Calculating the centripetal force:

$$\mathbf{F = M \cdot V^2 / R}$$

F: centripetal force, expressed in **N**
M: mass, expressed in **kg**
V: speed, expressed in $\mathbf{m.s^{-1}}$
R: radius of trajectory, expressed en **m**

consistency of the units: $\mathbf{F} = kg \cdot (m.s^{-1})^2 \cdot m^{-1} = kg \cdot (m^2.s^{-2} \cdot m^{-1}) = kg.m.s^{-2} = \mathbf{N}$

Let's calculate the centripetal force that drives the Earth around the Sun. Characteristics of the Earth: mass 6×10^{24} kg; orbital speed 30×10^3 $m.s^{-1}$; orbital radius 150×10^9 m.

$$F = 6 \times 10^{24} \times (30 \times 10^3)^2 / (150 \times 10^9)$$

$$F = 6 \times 10^{24} \times 900 \times 10^6 / (150 \times 10^9)$$

$$F = 6 \times 900 \times 150^{-1} \times 10^{+24} \times 10^{+6} \times 10^{-9}$$

$$\mathbf{F} = 36 \times 10^{21}\ \mathbf{N} = 36\ \mathbf{ZN}$$

Note 1: only one force can explain the circular motion of the Earth around the Sun, it is the centripetal force. This force is acting on the center of mass of the Earth and is oriented towards the center of the Sun. No other force exists.

Note 2: according to Isaac Newton's principle of reciprocal action, a reaction of same intensity but opposite direction is acting on the center of mass of the Sun, without effect on its own movement, because of its mass.

2. Calculating transverse accélération:

$$\mathbf{Y = F / M}$$

Y: transverse acceleration, expressed in $\mathbf{m.s^{-2}}$
F: centripetal force, expressed in **N**
M: mass, expressed in **kg**

consistency of the units: $\mathbf{Y} = kg.m.s^{-2} . kg^{-1} = \mathbf{m.s^{-2}}$

Let's calculate the transverse acceleration experienced by the Earth. Characteristics of the Earth: mass 6×10^{24} kg; centripetal force 36×10^{21} N.

$$Y = 36 \times 10^{21} / (6 \times 10^{24})$$

$$Y = 36 \times 6^{-1} \times 10^{+21} \times 10^{-24}$$

$$\mathbf{Y} = 6 \times 10^{-3} = 0.006\ \mathbf{m.s^{-2}}$$

B. Static description

The Earth stops turning around the Sun, it remains motionless in space.

Calcularing centrifugal force:

$$\mathbf{F' = -M . Y}$$

F': centrifugal force, expressed in **N**
M: mass, expressed in **kg**
Y: transverse acceleration, expressed in $\mathbf{m.s^{-2}}$

cohérence des unités : $\mathbf{F'} = kg . m.s^{-2} = \mathbf{N}$

Let us calculate the force that would act on the center of mass of the Earth, if it were motionless (zero orbital velocity), to maintain it in equilibrium in space and prevent it from falling towards the Sun:

$$F' = -6 \times 10^{24} \times 6 \times 10^{-3}$$

$$F' = -6 \times 6 \times 10^{+24} \times 10^{-3}$$

$$\mathbf{F'} = -36 \times 10^{21}\ \mathbf{N} = -36\ \mathbf{ZN}$$

Note 1: the sign [–] is required, it specifies the spatial orientation of this force conflicts the logic of the solar system.

Note 2: this force is commonly called '*centrifugal force*' which is incorrect since there is no orbital velocity, neither orbital radius nor center (the Earth is motionless and remains in equilibrium). The scientific name of this force is: imaginary force, fictional force, or pseudo-force.

VII. CALCULATION: THE CENTRIFUGE

Let us take as an example a centrifuge with a drum of 0.25 meters in radius which rotates at a speed of 1,200 rpm (20 revolutions per second).

1. Calculation of the circumferential velocity of the drum:

$$V = 2\,\pi .\ R .\ \omega$$

V: circumferential velocity, expressed in $\mathbf{m.s^{-1}}$
π: constant characteristic of the circle, equal to **3.14** (dimensionless)
R: radius of the drum, expressed in **m**
ω: speed of rotation, expressed in $\mathbf{tr.s^{-1}}$

coherence of units: $\mathbf{V} = m.\ s^{-1} = \mathbf{m.s^{-1}}$

$\mathbf{V} = 2 \times 3.14 \times 0.25 \times 20 = 31.4\ \mathbf{m.s^{-1}}$ = about 70 mph

This velocity, also called tangential velocity, is the *linear velocity* of drops of water when they leave the drum. Indeed, in the absence of atmosphere, gravitation and obstacle, drops of water would keep this speed on a straight path for eternity, hence the name of linear velocity.

2. Calculation of the transverse acceleration of the laundry:

$$Y = V^2 / R$$

Y: transverse acceleration, expressed in $\mathbf{m.s^{-2}}$
V: circumferential velocity, expressed in $\mathbf{m.s^{-1}}$
R: radius of trajectory, expressed in **m**

coherence of units: $\mathbf{Y} = (m.s^{-1})^2 .\ m^{-1} = (m^2.s^{-2},\ m^{-1}) = \mathbf{m.s^{-2}}$

$\mathbf{Y} = 31.4^2 / 0.25 = 986 / 0.25 = 4{,}000\ \mathbf{m.s^{-2}}$ = about 400 "**g**"

This transverse acceleration is oriented towards the axis of the drum, it keeps the laundry on a circular path, it comes from the *contact force* that the drum exerts on the laundry.

By virtue of the principle of reciprocity, the laundry exerts a reciprocal action on the drum, of equal intensity to this contact force, but of opposite direction. There is no centrifugal force, no centripetal force.

VIII. CALCULATION: A BALL THAT TURNS AROUND AN AXIS

A. Dynamic description

A ball of mass 100 grams (0.1 kg) weighing 1 N (gravitational acceleration "**g**" = 10 $m.s^{-2}$) is hanging from a string and rotates around an axis by describing a circular path of 0.25 radius meter at the speed of 1 turn per second.

1. Calculating the circumferential speed of the ball:

$$V = 2\,\pi\,.\,R\,.\,\omega$$

V: circumferential speed, expressed in **$m.s^{-1}$**
π: constant characteristic of the circle (3.14), without dimension;
R: radius, expressed in **m**
ω: revolutions per second, expressed in **$r.s^{-1}$**
consistency of the units: $\mathbf{V} = m\,.\,s^{-1} = \mathbf{m.s^{-1}}$

$\mathbf{V} = 2 \times 3.14 \times 0.25 \times 1 = 1.57\ \mathbf{m.s^{-1}}$

2. Calculating the horizontal force:

$$F = M\,.\,V^2 / R$$

F: horizontal force, expressed in **N**
M: mass, expressed in **kg**
V: circumferential speed, expressed in **$m.s^{-1}$**
R: radius, expressed in **m**
consistency of the units : $\mathbf{F} = kg\,.\,(m.s^{-1})^2\,.\,m^{-1} = kg\,.\,(m^2.s^{-2}\,.\,m^{-1}) = kg.m.s^{-2} = \mathbf{N}$

$\mathbf{F} = 0.1 \times 1.57^2 / 0.25 = 0.1 \times 2.465 / 0.25 = 0.986\ \mathbf{N}$

3. Calculating the tension of the string$^{(*)}$:

$$T = [P^2 + F^2]^{1/2}$$

T: string tension, expressed in **N**
P: ball weight, expressed in **N**
F: horizontal force, expressed in **N**
consistency of the units : $\mathbf{T} = [(kg.m.s^{-2})^2]^{1/2} = [kg^2.m^2.s^{-4}]^{1/2} = kg.m.s^{-2} = \mathbf{N}$

Note: ½ power equals a square root.

$\mathbf{T} = [1^2 + 0.986^2]^{1/2} = [1 + 0.972]^{1/2} = 1.972^{1/2} = 1.4\ \mathbf{N}$

B. Static description

Photograph of a 100 gram (0.1 kg) ball rotating around an axis shows that the string holding the ball forms an angle of 44.6 degrees respective to the vertical (gravitational acceleration "**g**" = 10 $m.s^{-2}$).

The movement being frozen, it is necessary to imagine a force acting on the center of mass of the ball to maintain it in equilibrium.

This imaginary force is the centrifugal force, which will be called for want of better (the ball being motionless, it does not move away, moreover, there is no circle or center).

Calculating the centrifugal force:

$$F' = - M \cdot g \cdot \tan \alpha$$

F': centrifugal force, expressed in **N**
M: mass, expressed in **kg**
g: gravitational acceleration, expressed in $m.s^{-2}$
α: angle to the vertical, dimensionless;
consistency of the units : **F'** = kg . $m.s^{-2}$ = **N**

Note: Trigonometric values are dimensionless.

F' = – 0.1 x 10 x tangent 44.6° = – 1 x 0.986 = – 0.986 **N**

Warning! The sign [–] is required, it means that the spatial orientation of the centrifugal force conflicts the logic of the movement.

At last, it should be noted that, in the context of this description, this calculation is the only one allowed. Indeed, it is impossible to evoke the radius of the trajectory of the ball, let alone its speed!

(*)*The tension of the string is the result of the weight and the horizontal force, it is equal to the vector sum of these two forces (green arrow on the drawing on the left, page 11). The vector sum is calculated from the right triangle formed by the translation of one of the two vectors (without modifying its length or its orientation), according to the properties of the right triangles discovered by Pythagoras, Greek philosopher and mathematician of the 5th century before our era. Pythagoras' theorem states that, in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. In other words, in a right triangle, the length of the hypotenuse is equal to the square root of the sum of the squares of the other two sides, this is the relation used here.*

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