

THE KINETIC ENERGY

All the experts agree that one of the greatest difficulties of driving is to properly estimate the speed, but also and above all to evaluate the speed differences.

Where does this difficulty come from?

The sensory organs or perceptive sensors that are the eyes are not measuring instruments per se, they only can inform us in a rather subjective manner on movement and speed.

That is why all cars are equipped with a speedometer so that the driver can have at any time an objective data for driving.

However, being informed of the exact speed is one thing, being able to grasp the speed differences is another. Indeed, automotive phenomena such as the fuel consumption, the length of the braking distance or the consequences of possible collisions are always associated with variations of kinetic energy rather than simple speed differences.

In other words, only the calculation allows a real and accurate assessment of these differences, better than the speedometer that will be useless. Here are the details of this reasoning with unexpected consequences.

Defining kinetic energy

Energy refers to any manifestation of movement, heat or radiation. The energy that appears as motion is called kinetic energy.

There are three more precise definitions of the kinetic energy, all three completely equivalent under the general law of conservation of energy discovered and formulated by the English physicist James Joule^(*).

First definition: the kinetic energy is the amount of energy used to " make " the movement. Indeed, in the absence of energy, a mass remains motionless or maintains a constant speed. The kinetic energy is an accurate indication of the energy used to accelerate a mass, minus any loss due to heat, friction and various resistances.

Second definition: the kinetic energy is the amount of energy that must be entirely dissipated for complete immobilization of a moving mass. Kinetic energy is therefore an accurate indication of the difficulty to decelerate a mass.

Third definition: the kinetic energy is the amount of energy dissipated in a collision. The kinetic energy is an accurate indication of the violence of an impact.

Expressing the kinetic energy

Kinetic energy is a scalar quantity, without spatial orientation, it can only be represented by a number. It is expressed by the equation:

$$E = \frac{1}{2} M V^2$$

According to the International System of Units (**SI**) whose use is, let us recall, mandatory everywhere in the world regardless of the field concerned (industry, trade, education), the mass is expressed in kilograms (symbol **kg**), the speed in meters per second (symbol **m.s⁻¹**) and the kinetic energy in joules (symbol **J**) in tribute to the work of James Joule.

The consistency check of the units is written as:

$$\text{Kinetic energy} = \text{kg} \cdot (\text{m} \cdot \text{s}^{-1})^2 = \text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2} = \text{joule}$$

Example: let us calculate the kinetic energy of a mass 3,300 lb (1,500 kg) traveling at 67 mph speed (30 meters per second)^(**):

$$E = \frac{1}{2} M V^2$$

$$E = \frac{1}{2} \times 1,500 \times 30^2$$

$$E = 750 \times 900 = 675,000 \text{ J}$$

Creating or removing energy

It is nothing miraculous about energy: creating, reducing or removing kinetic energy requires physical forces.

In the case of the car and under normal circumstances, these forces act on the tires in contact with the ground. But when a car collides with a tree or a wall, this force comes from the obstacle and acts on the body of the car.

The work done by these forces matches exactly the amount of kinetic energy stored or dissipated.

Relationships between parameters

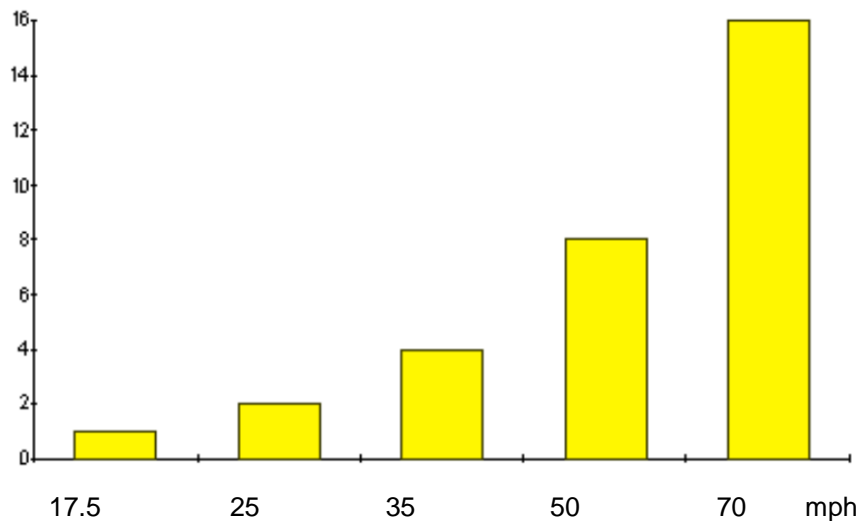
Let us look at relationships between physical parameters. What does the formula of

the kinetic energy tell us? The mass is obviously a constant, while the squared speed is the variable.

In practical terms, that means that the kinetic energy is not proportional to the speed but to the square of the speed.

In other words, if the speed is doubled, the kinetic energy is multiplied by four; if the speed is tripled, the kinetic energy is multiplied by nine; if the speed is quadrupled, the kinetic energy is multiplied by sixteen, etc.

It can also be concluded that the kinetic energy is doubled when the speed is only multiplied by a factor of 1.414 as $2^{1/2} = 1.414$.



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Diagram of the kinetic energy as a function of speed.

The role of the speed

The relationship between energy and speed shows us that:

- to double the speed of a car, you need to consume four times more fuel^(**); the braking distance is then four times longer and, in case of collision, the impact is four times more violent!

- to triple the speed of a car, you need to consume nine times more fuel^(**); the braking distance is then nine times longer and, in case of collision, the impact is nine times more violent!

- to quadruple the speed of a car, you need to consume sixteen times more fuel^(**); the braking distance is sixteen times longer and, in case of collision, the impact is sixteen times more violent! etc.

Therefore, now we sense the interest of more accurately calculate what actually represent these speed differences, especially since they seem insignificant to us. How to do it?

Kinetic energy variation

Any speed variation means kinetic energy variation:

- to increase car speed, you have to give it kinetic energy;
- to reduce car speed, you have to dissipate kinetic energy.

The variation of kinetic energy is expressed thanks to the relation:

$$\Delta E = \frac{1}{2} M (Vb^2 - Va^2)$$

In accordance with the International System of Units, the mass is expressed in kilograms (symbol kg), Va denotes the initial speed and Vb the gained speed, both expressed in meters per second (symbol $m.s^{-1}$). The kinetic energy is expressed in joule (symbol J). The verification of the consistency of the units:

$$\Delta E = kg \cdot [(m.s^{-1})^2 - (m.s^{-1})^2] = kg \cdot (m.s^{-1})^2 = kg.m^2.s^{-2} = \text{joule.}$$

Example: Let's calculate the kinetic energy variation of a mass 3,300 lb (1,500 kg) when speed goes from 33.5 mph ($15 m.s^{-1}$) to 67 mph ($30 m.s^{-1}$):

$$\Delta E = \frac{1}{2} M (Vb^2 - Va^2)$$

$$\Delta E = \frac{1}{2} \times 1,500 \times (30^2 - 15^2)$$

$$\Delta E = 750 \times (900 - 225)$$

$$\Delta E = 750 \times 675 = 506,250 \text{ J}$$

Correspondence between energy variation and speed

Knowing the mass of the car and the variation of energy, it is easy to calculate the corresponding speed. To do this, simply reverse the relationship:

$$E = \frac{1}{2} M V^2$$

which implies:

$$V = (2 E / M)^{1/2}$$

Example: let's calculate the speed that corresponds to a kinetic energy of 506,250 J for mass 1,500 kilograms:

$$V = (2 \times 506,250 / 1,500)^{1/2}$$

$$V = (1,012,500 / 1,500)^{1/2}$$

$$V = 675^{1/2} = 26 \text{ m.s}^{-1} = 58 \text{ mph}$$

This result means that to vary the speed of from 33.5 mph (15 m.s⁻¹) to 67 mph (30 m.s⁻¹), it is necessary to communicate to it a quantity of energy equivalent to that this car possesses when it circulates at 58 mph.

In other words, speeding up a car from 33.5 mph to 67 mph is, from the point of view of energy, like giving to it the equivalent of a speed of 58 mph.

A simplified formula ...

The mass being constant, the variation of kinetic energy only depends on the square of the speed. We can test it by considering a two-kilogram mass.

And since this is not about calculating an amount, but about evaluating a difference impacting the variable, it is then possible to calculate the difference with the speed expressed in miles per hour (symbol mph).

In other words, the simple calculation of the square root of the difference of squares of the speed expressed in miles per hour tells us correctly and precisely about the variation of kinetic energy, and thus the corresponding difference in speed!

The simplified relationship is then:

$$V = (Vb^2 - Va^2)^{1/2}$$

In this relationship, V is the variation of kinetic energy between the speed Vb and the speed Va, all these three magnitudes expressed in miles per hour. The power $\frac{1}{2}$ means the square root of this difference.

The relationship fulfills the consistency requirement of the units, which is proved by the following:

$$\text{variation of kinetic energy} = [(\text{mph})^2]^{1/2} = \text{mph}$$

Not only the calculation is faster, but more importantly the result is particularly meaningful, as we shall see in the following examples.

Some practical examples ...

Readers familiar with calculations can, using the complete formula expressing the kinetic energy, calculate it for each example provided and thus check the results indicated.

First example: what is the difference of kinetic energy between 30 and 40 mph?

$$V = (40^2 - 30^2)^{1/2} = (1,600 - 900)^{1/2} = (700)^{1/2} = 26.5 \text{ mph}$$

It can be inferred the following: to accelerate from 30 to 40 mph, you must consume as much fuel^(**) as to accelerate from 0 to 26.5 mph; to brake from 40 to 30 mph, it is necessary to dissipate as much energy as for braking from 26.5 to 0.

From the energy point of view, driving at 40 mph is like adding 26.5 mph to 30 mph!

Second example: what is the difference of kinetic energy between 50 and 60 mph?

$$V = (60^2 - 50^2)^{1/2} = (3,600 - 2,500)^{1/2} = (1,100)^{1/2} = 33.2 \text{ mph}$$

It can be inferred the following: to accelerate from 50 to 60 mph, you must consume as much fuel^(**) as to accelerate from 0 to 33.2 mph; to brake from 60 to 50 mph, it is necessary to dissipate as much energy as for braking from 33.2 to 0.

From the energy point of view, driving at 60 mph is like adding 33.2 mph to 50 mph!

Third example: what is the difference of kinetic energy between 70 and 80 mph?

$$V = (80^2 - 70^2)^{1/2} = (6,400 - 4,900)^{1/2} = (1,500)^{1/2} = 38.7 \text{ mph}$$

It can be inferred the following: to accelerate from 70 to 80 mph, you must consume as much fuel^(**) as to accelerate from 0 to 38.7 mph; to brake from 80 to 70 mph, it is necessary to dissipate the same energy than for braking from 38.7 to 0.

From the energy point of view, driving at 80 mph is like adding 38.7 mph to 70 mph!

A table to sum up...

The following table includes all the values calculated from 15 mph for the usually practiced speeds in the United States.

75	73.5	72.3	71	68.7	66	63.4	60	56	51	45	37.4	27	0
65	63	62	60	57.7	54.8	51.2	47	41.5	34.6	25	0	26	37.4
55	53	51	49	46	42.4	37.7	31.6	23	0	24	34.6	43	51
45	42.4	40.3	37.4	33.5	28.3	20.6	0	21.8	31.6	39.7	47	53.6	60
35	31.6	28.7	24.5	18	0	19.4	28.3	35.7	42.4	48.7	54.8	60.6	66
25	20	15	0	16.6	24.5	31	37.4	43.3	49	54.5	60	65.4	71
15	0	13.2	20	26	31.6	37	42.4	47.7	53	58	63	68.4	73.5
mph	15	20	25	30	35	40	45	50	55	60	65	70	75

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How to read this chart? You just need to match the speed shown in bold in the left column and bottom line. The value shown is the difference of kinetic energy expressed in miles per hour.

Example 1: between 15 and 25 mph, the difference is 20 mph. This means that for a car to accelerate from 15 to 25 mph, it needs as much energy as to accelerate from 0 to 20 mph, the difference being the same in braking or in a collision.

Example 2: between 25 and 55 mph, the difference is 49 mph. This means that for a car to accelerate from 25 to 55 mph, it needs as much energy as to accelerate from 0 to 49 mph, the difference being the same in braking or in a collision.

Example 3: between 55 and 75 mph, the difference is 51 mph. This means that for a car to accelerate from 55 to 75 mph, it needs as much energy as to accelerate from 0 to 51 mph, the difference being the same in braking or in a collision.

Such differences are obviously impossible to estimate only from the data displayed on the speedometer, let alone the driver's perceptions or feelings...

A law of unexpected consequences ...

A consequence of this law is that the higher the speed, the greater the differences!

Translated into current arithmetic language, this law could be written as:

$$25 \text{ mph} = 15 \text{ mph} + 20 \text{ mph}$$

$$55 \text{ mph} = 25 \text{ mph} + 49 \text{ mph}$$

$$75 \text{ mph} = 55 \text{ mph} + 51 \text{ mph}$$

Or even:

$$75 \text{ mph} = 15 + 20 + 49 + 51 \text{ mph!}$$

This equivalence can be verified as follows:

$$75^2 = 15^2 + 20^2 + 49^2 + 51^2 = 5,625^{(***)}$$

Concretely, this means that only one car traveling at 75 mph has as much kinetic energy as four identical cars circulating respectively at 15, 20, 49 and 51 mph!

Therefore, we now understand that the requirement of road safety, of energy saving and environmental protection, command to limit the speed to reasonable levels.

(*) *James Prescott Joule, English physicist (1818-1889). The law of conservation of energy states that the work, heat and energy are equivalent quantities that change in shape but are still fully conserved.*

(**) *This calculation takes into account only the energy necessary to accelerate a mass in a rectilinear motion, this is called the translational kinetic energy. The total quantity is greater if we consider, on the one hand, the rotational kinetic energy which is defined as the energy necessary to create the rotation of the engine and the transmission and the wheels, on the other hand, the action of resistant forces (rolling resistance, air resistance) during the period of time that this acceleration lasts. The rotational kinetic energy of the four wheels of a passenger car represents about 5 % of the translational kinetic energy, this proportion remaining constant whatever the speed.*

(***) *The correct result is 5,627, the difference being due to approximations of exact square roots.*

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SOME RELATIONSHIPS BETWEEN PHYSICAL QUANTITIES

Kinetic energy (translatory movement)

$$E = \frac{1}{2} M \cdot V^2$$

E: kinetic energy, expressed in **J**

M: mass, expressed in **kg**

V: speed, expressed in **m.s⁻¹**

consistency of units: $E = \text{kg} \cdot (\text{m} \cdot \text{s}^{-1})^2 = \text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2} = \text{J}$

Example: let's calculate the kinetic energy of a car weighing 3,300 lb (1,500 kg) at 45 mph (20 m.s⁻¹):

$$E = \frac{1}{2} \times 1,500 \times 20^2 = 750 \times 400 = 300,000 \text{ J}$$

Kinetic energy (rotary movement)

$$E = \frac{1}{2} M \cdot R^2 \cdot \omega^2$$

E: kinetic energy, expressed in **J**

M: mass, expressed in **kg**

R: radius, expressed in **m**

ω: rotational speed, expressed in **rad.s⁻¹**

consistency of units: $E = \text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2} = \text{J}$

(radian is dimensionless quantity)

Example: calculate the rotary kinetic energy of the four wheels of a car, each wheel assimilated to a solid and homogeneous disk of mass 18 kg and of radius 0.30 m with a rotational speed of 10 revolutions per second (62.8 rad.s⁻¹):

$$E = (\frac{1}{2} \times 18 \times 0.30^2 \times 62.8^2) \times 4 = (9 \times 0.09 \times 3,950) \times 4 = 3,200 \times 4 = 12,800$$

Total kinetic energy

$$E_{(\text{total})} = E_{(\text{translatory})} + E_{(\text{rotary})}$$

Example: calculate total kinetic energy of a car, the translatory one being 300 kJ, the four wheels rotary one being 15 kJ (disregarding movement of the engine, gearbox and transmission shafts):

$$E_{(\text{total})} = 300 + 15 = 315 \text{ kJ}$$

Kinetic energy variation

$$\Delta E = \frac{1}{2} M \cdot (V_a^2 - V_b^2)$$

ΔE : kinetic energy variation, expressed in **J**

M: mass, expressed in **kg**

V_a: initial speed, expressed in **m.s⁻¹**

V_b: gained speed, expressed in **m.s⁻¹**

consistency of units: $E = \text{kg} \cdot [(\text{m} \cdot \text{s}^{-1})^2 - (\text{m} \cdot \text{s}^{-1})^2] = \text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2} = \text{J}$

Example: let's calculate the kinetic energy variation of a car weighing 3,300 lb (1,500 kg) when the speed goes from 67 mph (30 m.s⁻¹) to 33.5 mph (15 m.s⁻¹) (disregarding movement of the engine, gearbox, transmission shafts and wheels):

$$\Delta E = \frac{1}{2} \times 1,500 \times (30^2 - 15^2) = 750 \times (900 - 225) = 750 \times 675 = 506,250 \text{ J}$$

Energy and speed equivalence

$$V = (2 E / M)^{1/2}$$

V: speed, expressed in **m.s⁻¹**

E: kinetic energy, expressed in **J**

M: mass, expressed in **kg**

consistency of units: $V = (\text{kg}^{+1} \cdot \text{m}^{+2} \cdot \text{s}^{-2} \cdot \text{kg}^{-1})^{1/2} = (\text{m}^{+2} \cdot \text{s}^{-2})^{1/2} = \text{m} \cdot \text{s}^{-1}$

Example: Let's calculate the speed of a car weighing 3,300 lb (1,500 kg) that equals a 506,250 J energy (disregarding movement of the engine, gearbox, transmission shafts and wheels):

$$V = (2 \times 506,250 / 1,500)^{1/2} = (1,012,500 / 1,500)^{1/2} = (675)^{1/2} = 26 \text{ m} \cdot \text{s}^{-1} = 58 \text{ mph}$$

Simplified formula

$$V = (V_b^2 - V_a^2)^{1/2}$$

V: speed, expressed in **mph**

V_a: initial speed, expressed in **mph**

V_b: gained speed, expressed in **mph**

consistency of units: $V = [(\text{mph})^2]^{1/2} = \text{mph}$

Example: calculate the speed corresponding to a kinetic energy variation between 67 mph and 33.5 mph (disregarding movement of the engine, gearbox, transmission shafts and wheels):

$$V = (67^2 - 33.5^2)^{1/2} = (4,489 - 1,122.25)^{1/2} = 3,367^{1/2} = 58 \text{ mph}$$

Total energy produced by fuel combustion

Diesel (density 845 kg.m^{-3}): **44.3 MJ.kg⁻¹** (**37.4 MJ.l⁻¹**)

Gasoline (density 760 kg.m^{-3}): **46.9 MJ.kg⁻¹** (**35.6 MJ.l⁻¹**)

LPG (density 550 kg.m^{-3}): **48.7 MJ.kg⁻¹** (**26.8 MJ.l⁻¹**)

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