THE SLOPES

The cycling Tour de France is the third largest sporting event in the world after the Olympic Games and the Football World Cup. Where does the attraction to this event and the popularity of cycling? Just ride a bicycle uphill to understand!

Definitions

The *altitude* is the vertical distance measured from sea level. The *elevation change* is the difference in altitude between two places.

The *gradient* is the ratio between the elevation change and the length of a road.

Expression of the gradient

The gradient can be expressed either as a percentage or a fraction or a decimal number generally between 0 and 0.25 for paved roads(1).

Example 1: a 10 % gradient (1/10 or 0.1) means that the elevation change is 10 meters over 100 meters traveled.

Example 2: the road from Malaucène to the top of Mont Ventoux (France, PACA region) has a length of 21 km and an elevation change of 1,530 meters; its average gradient is thus 7.3 % (73/1000 or 0.073).

Warning! Be careful not to confuse the gradient and the road's angle respect to the horizontal: a 10 % gradient does not mean that this angle is 10 degrees! The corresponding numerical values are:

gradient	2 %	4 %	6 %	8 %	10 %	12 %	14 %	16 %	18 %	20 %
angle	1.15°	2.29°	3.44°	4.6°	5.7°	6.9°	8°	9.2°	10.4°	11.5°

And conversely:

angle	2°	4°	6°	8°	10°	12°	14°	16°	18°	20°
gradient	3.5 %	7 %	10.5 %	13.9 %	17.4 %	20.8 %	24.2 %	27.6 %	30.9 %	34.2 %

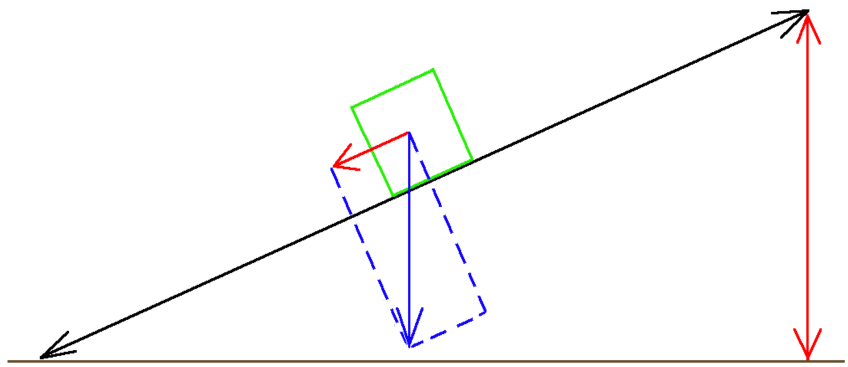
From the mathematical point of view, the gradient is the sine of the angle formed by the road respect to the horizontal, to distinguish the gradient at geometric sense, which is the tangent of the angle, even if the difference between these two values is negligible on the road network.

The component parallel to the road weight

What about the movement of land vehicles if the road is not perfectly flat?

Due to the slope, the weight has a component parallel to the road whose intensity is proportional to the sine of the angle formed by the road relative to the horizontal[(2)].

This force creates a downhill movement, it opposes the uphill movement.



Component of the weight on a slope (red arrow).

Slope and traction force

The traction force is defined as the force exerted on contact with the ground by the engine to create or maintain the movement of land vehicles (see ADILCA file '*engine torque*').

The gradient gives immediate information about the traction force to maintain the uphill movement.

Indeed, disregarding both resistive forces (rolling resistance, air resistance), the traction force to maintain a constant speed must be strictly equal and opposite to the component of the weight parallel to the road.

Example 1: a 100 kg mass cyclist (bike included) traveling a 10 % uphill gradient will apply a traction force of about 100 N, or 10 % of its weight ('**g**' ~ 10 $m.s^{-2}$).

Example 2: a 1,500 kilograms mass car traveling a 10 % uphill gradient will apply a traction force of about 1,500 N, or 10 % of its weight ('**g**' ~ 10 $m.s^{-2}$).

Example 3: a 40-ton truck traveling a 10 % uphill gradient will apply a traction force of about 40,000 N, or 10 % of its weight ('**g**' ~ 10 $m.s^{-2}$).

Slope and engine brake force

The engine brake force is defined as an opposite traction force that slows the car when the driver releases the accelerator (see ADILCA file '*engine torque*').

Such a force is mainly used to stabilize the speed downhill, natural resistances (rolling resistance, air resistance) being too weak to be sufficient.

Example: a 1,500 kilograms mass car traveling a 10 % downhill gradient will apply an engine brake force of about 1,500 N, or 10 % of its weight ('g' ~ 10 $m.s^{-2}$).

Slope and braking force

The brake force is defined as an opposite force that slows the car when the driver presses on the brake pedal (see ADILCA file '*braking force*'). Be careful not to confuse engine brake force and braking force!

The component of the weight parallel to the road is added to the braking force uphill, it is subtracted from the braking force downhill.

Example: a car that can apply a braking force of 15,000 N on a flat road can apply, all other conditions being equal, a total braking force of approx. 16,500 N on a 10 % uphill gradient, about 13,500 N on a 10 % downhill gradient ('**g**' ~ 10 $m.s^{-2}$).

Slope and work

The work in the physical sense of the term, is defined as the energy required for moving a force.

Disregarding air resistance and rolling resistance, assuming a perfect transmission efficiency of the bicycle, the work of a cyclist on a constant slope depends only on the distance traveled[3].

Example 1: if we consider a cyclist mass 100 kg (bicycle included) traveling a 10 % uphill gradient over 10 km, the work is about 1 MJ and therefore mobilizes equivalent muscular energy.

Example 2: if we consider a car mass 1,500 kg traveling a 10 % uphill gradient over 10 km, the work is about 15 MJ and therefore mobilizes equivalent motive energy.

Slope and power

The power is defined as the work done per unit of time.

Disregarding air resistance and rolling resistance, assuming that the transmission efficiency of the bicycle is total, the power only depends on the speed with which the work is accomplished.

Example 1: if we consider a cyclist mass 100 kg (bicycle included) traveling a 10 % uphill gradient at a constant speed of 4 $m.s^{-1}$ (9 mph), the power is about 400 W, a value well beyond the physical capacity of a normal individual.

Indeed, although some very talented and well-trained cyclists are able to temporarily deliver a power of 450 W, it is assumed that an effort of endurance achieved by a normal individual should not mobilize a power greater than 100 W.

Example 2: if we consider a car mass 1,500 kg traveling a 10 % uphill gradient at a constant speed of 20 $m.s^{-1}$ (45 mph), the engine power required for this slope is about 30 kW (41 hp).

In reality, much greater power is required due to the rolling resistance and air resistance[(4)].

Gravitational energy

Gravitational energy is defined as the energy accumulated by a mass falling in free fall (disregarding the air resistance), or as the energy needed to lift a mass vertically using a winch (disregarding friction), these two quantities being strictly equal.

That is very convenient for summarizing slope problems. Indeed, some quick calculations make it possible to verify that, whatever the path taken, the work of a traction force uphill (or engine brake force downhill) and the gravitational energy are two equivalent quantities, provided to disregard the two natural resistances which can affect the movement of land vehicles (rolling resistance and air resistance).

It is deduced that the work (in the physical sense of the term), and therefore the energy and the power depend only on the elevation change, in other words on the altitude's difference.

Altitude and Weather

Weather conditions (atmospheric pressure, temperature) vary with altitude. This is because the air density decreases gradually as one moves away from the ground. And when the density of air decreases, the temperature drops.

Considering the characteristics of a vertical column of air, the air density at 1,000 meters height is 12 % smaller than at sea level.

At 2,000 meters height, the air density is 21 % smaller than at sea level; at 3,000 meters height, it is 29 % smaller than at sea level.

If the temperature of the air column is 20 °C (70 °F) at sea level, it is 15 °C (60 °F) at 1,000 meters height, 10 °C (50 °F) at 2,000 meters height, 5 °C (40 °F) at 3,000 meters height, etc.

For cyclists that evolve in the mountains, these factors must be taken into account. The most immediate effect is the feeling of suffocation, as the filling of the lungs becomes more difficult.

In addition, in equal volumes, the mass of absorbed air with each breath is reduced, that decreases muscle performance accordingly, especially as the body must also compensate for lower ambient temperature.

Therefore, except for acclimatization to increase the number of red blood cells responsible for precious oxygenation, physical efforts in the mountains are always very demanding and we must be careful not to compare sport performances in altitude with those completed at sea level.

Of course, these physical realities have the same consequences on the operation of automobile engines since they need air to burn fuel (see ADILCA file '*fuel combustion*').

(1)*The Alto de Angliru, located in the Spanish province of Asturias, is considered as one of the steepest mountain road in the world with 23 % gradient.*

(2) *The weight varies according to the latitude, because of the particular form of the terrestrial globe, flattened at the poles and swollen with the equator (the gravitational acceleration is inverse function of the square of the distance which separates from the center of the Earth). On the other hand, over a given geographical area, the variation of weight is negligible up to 5,000 m of altitude (the highest roads of Europe do not reach 3,000 meters: Stelvio pass in Italy, at 2,758 m altitude, Iseran pass in France, at 2,770 m altitude).*

(3) *The transmission of a bicycle absorbs a little energy: rotation of the pedals, rotation of crankshaft, movement of the chain, rotation of the links of the chain and rotation of the wheels of the derailleur.*

(4) *A 1,500-kilogram mass car traveling uphill a 10 % gradient at a speed of 20 $m.s^{-1}$ (45 mph) must apply a traction force of about 1,500 N to balance the weight component and maintain its speed. If we add the rolling resistance (about 120 N at this speed) and the air resistance (about 180 N at this speed), the effective power that the engine must deliver to maintain a constant speed is about 36 kW (50 hp). A good precaution consisting of not soliciting more than 50 % of the possibilities of an engine, it means that, to achieve such a performance without mechanical risk, this engine must deliver a maximum power of about 150 hp (see ADILCA files* 'engine torque' *and* 'engine power'*).*

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SOME RELATIONSHIPS BETWEEN PHYSICAL QUANTITIES

Weight

$$\mathbf{P = M \cdot g}$$

P: weight, expressed in **N**
M: mass, expressed in **kg**
g: gravitational acceleration, expressed in $\mathbf{m.s^{-2}}$
(Earth : **g** = 9.8 $m.s^{-2}$)
consistency of the units: **P** = kg . $m.s^{-2}$ = **N**

Example: calculate the weight of a 1,500 kg mass car:

P = 1,500 x 9.8 = 14,700 N

Gradient

$$\mathbf{\alpha = H / L}$$

α: gradient, dimensionless ;
H: elevation change, expressed in **m**
L: length of the road, expressed in **m**
consistency of the units: **α** = m^{+1} . m^{-1} = dimensionless.

Example: calculate the gradient of a 10 kilometers (6.2 miles) road with a 1,000 meters (3,000 feet) elevation change:

α = 1,000 / 10,000 = 0.1 = 1/10 = 10 %

Component of the weight

$$\mathbf{F = M \cdot g \cdot \alpha}$$

F: component of the weight, expressed in **N**
M: mass, expressed in **kg**
g: gravitational acceleration, exprimée en $\mathbf{m.s^{-2}}$
α: gradient, dimensionless ;
consistency of the units: **F** = kg . $m.s^{-2}$ = **N**

Example: calculate the component of the weight of a 1,500 kg (3,300 lb) mass car on a 0.1 gradient slope (**g** = 9.8 $m.s^{-2}$):

F = 1,500 x 9.8 x 0.1 = 1,470 N

Traction work

$$\mathbf{E = F \cdot D}$$

E: work, expressed in **J**
F: component of the weight, expressed in **N**
D: distance, expressed in **m**
consistency of the units: $\mathbf{E} = N \cdot m = kg.m^{+1}.s^{-2} \cdot m^{+1} = kg.m^{2}.s^{-2} = \mathbf{J}$

Example: calculate the traction work of a 1,470 N traction force traveling 10 kilometers (6.2 miles):

$$E = 1{,}470 \times 10{,}000 = 14{,}700{,}000 \text{ J}$$

Gravitational energy

$$\mathbf{E = M \cdot g \cdot H}$$

E: energy, expressed in **J**
M: mass, expressed in **kg**
g: gravitational acceleration, expressed in $\mathbf{m.s^{-2}}$
H: elevation change, expressed in **m**
consistency of the units: $\mathbf{E} = kg \cdot m^{+1}.s^{-2} \cdot m^{+1} = kg.m^{2}.s^{-2} = \mathbf{J}$

Example 1: calculate the gravitational energy of a 1,500 kg (3,300 lb) mass with a 1,000 meters elevation change ($\mathbf{g} = 9.8\ m.s^{-2}$):

$$E = 1{,}500 \times 9.8 \times 1{,}000 = 14{,}700{,}000 \text{ J}$$

Example 2: calculate the gravitational energy of a 1,500 kg (3,300 lb) mass falling free from 1,000 meters height ($\mathbf{g} = 9.8\ m.s^{-2}$):

$$E = 1{,}500 \times 9.8 \times 1{,}000 = 14{,}700{,}000 \text{ J}$$

Power absorbed by slope

$$\mathbf{B = F \cdot V}$$

B: power, expressed in **W**
F: component of the weight, expressed in **N**
V: speed, expressed in $\mathbf{m.s^{-1}}$
consistency of the units: $\mathbf{B} = kg.m^{+1}.s^{-2} \cdot m^{+1}.s^{-1} = kg.m^{2}.s^{-3} = \mathbf{W}$

$$B = M \cdot g \cdot H / T$$

B: power, expressed in **W**
M: mass, expressed in **kg**
g: gravitational acceleration, expressed in $m.s^{-2}$
H: elevation change, expressed in **m**
T: duration, expressed in **s**
consistency of the units: **B** $= kg \cdot m^{+1}.s^{-2} \cdot m^{+1} \cdot s^{-1} = kg.m^2.s^{-3} =$ **W**

Example 1: calculate the power absorbed by the slope, the component of the weight being a 1,470 N force at a speed of 20 $m.s^{-1}$ (45 mph):

$$B = 1{,}470 \times 20 = 29{,}400 \text{ W}$$

Example 2: calculate the power to climb a 1,500 kg mass to 1,000 m height in 10 minutes (600 s):

$$B = 1{,}500 \times 9.8 \times 1{,}000 / 600 = 24{,}500 \text{ W}$$

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