STATICS AND DYNAMICS AND CENTRIFUGAL FORCE

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I. THE LAWS OF NEWTON

The general laws of movement were discovered and formulated by the English mathematician and physicist Isaac Newton (1642 - 1727).

These laws are universal and allow you to describe any form of movement.

Being a circular movement, these laws read as follows:

Principle of rectilinear inertia

'*A moving mass on which no force acts, describes a perfectly rectilinear trajectory.*'

The concept of force stems from this principle.

Force concept

'*A force refers at any cause which deviates the trajectory of a mass.*'

Principle of reciprocity

'*Any mass subjected to a force, responds by a reciprocal action of equal intensity, but of opposite direction*.'

How do these laws apply in the case of a land vehicle in motion on a circular path, and how these laws work with statics and the concept of centrifugal force?

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II. STATIC AND DYNAMIC PHYSICAL LAWS

Does the vector derivation by transforming a rotating base into a fixed one (called the 'Frenet base') provide the mathematical proof of the existence of the centrifugal force? That's what we're going to check!

Some useful information

- A system of reference is a set of benchmarks to measure the characteristics of the movement of a mass. There are two types of systems: the absolute reference to describe a real movement, and the relative reference to describe an apparent motion.

The Earth is the absolute reference that allows the description of the movement of the cars and all they contain (passengers, luggage) because, as the name implies, land vehicles move relative to the Earth. On the opposite, a car can only be a relative reference that does not allow other description than that of the movement of passengers and luggage. This distinction was detailed in a folder to read elsewhere (see ADILCA folders).

- A force is any cause able to change the speed or the trajectory of a body. There are two types of forces: the real forces and imaginary forces.

The real forces act remotely or by contact, they are at the origin of the movement observed in an absolute reference. There are only two forces acting at a distance: the gravitational force and the electromagnetic force. By cons, there are a multitude of contact forces, including those governing the movement of land vehicles. The contact forces are acting on the tires in contact with the ground and they are four in number (driving force, engine brake, braking force and guiding force).

The imaginary forces (also called apparent forces, fictive forces or pseudo-forces), are of a different nature: they are supposed to be acting on the center of mass of a mass to explain apparent movement or balance but can only appear in a relative reference or in a static description. Physics knows only three imaginary forces: inertial force, centrifugal force and Coriolis force.

From reality to fiction

What is the transformation of a rotating base into a fixed base?

Mathematics and physics are certainly complementary, but we must avoid aligning calculations without considering their real meaning. In this case, the transformation of a rotating base into a fixed base ignores the real movement. It is a purely imaginary projection. In other words, the rotation base is then considered to be motionless. Then, it is necessary to resort to imaginary forces to explain an apparent equilibrium.

The conditional

How to separate reality from fiction? Here is an example:

- Sentence 1: I won the lottery \$ 300 and my grandmother left me \$ 700, I have \$ 1,000.

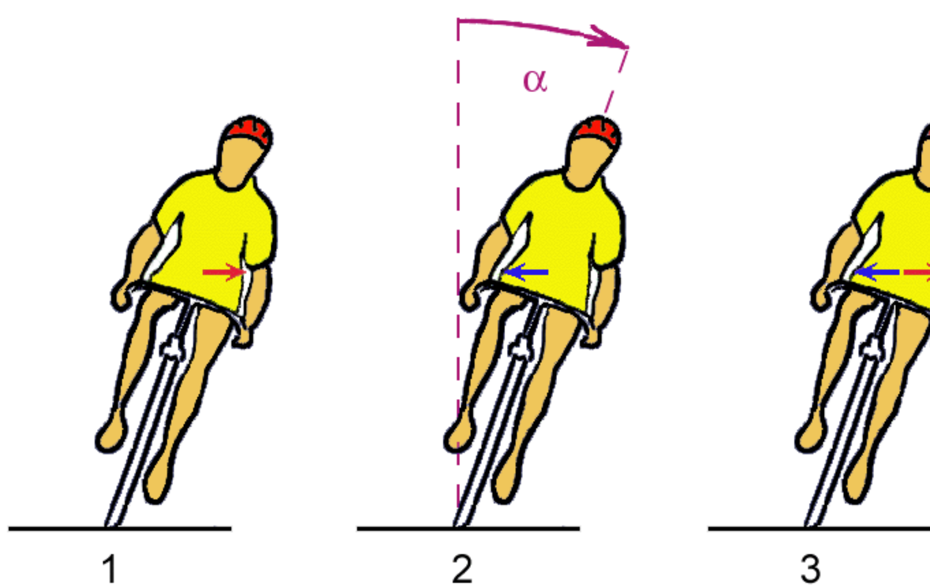
- Sentence 2: If I won the lottery \$ 300, and if my grandmother gave me \$ 700, I would have \$ 1,000.

We see that the two calculations are perfectly right in both cases, with one difference, however: the money is real only in the first sentence, it is completely fictive in the second sentence! These are the modes of conjugation that distinguish them, and there is no question of mixing them, neither from grammatical point of view nor from accounting point of view.

From dynamic to static

The same transformation is allowed as much in mathematics as in physics! Moving from a rotating base to a fixed base is like using this conditional, assuming that the real movement is frozen. The first description is named '*dynamic*', this is the real description. The second is called '*static*', this is an imaginary description. But better than words, here is the example of the cyclist to illustrate this distinction.

Three drawings better than a long speech



- Drawing 1: this is the real description, known as '*dynamic*'. The rotating base is made of a cyclist on his bicycle. Concrete translation: the cyclist is moving and describes a circle. According to Newton's law(*), the cyclist has been deviated from a straight path by a contact force F, unique and real, which causes the transverse acceleration of its center of mass, this is the *guiding force* (arrow red on the drawing 1). In reality this force is acting in contact with the ground but, for better readability of the drawing, we 'transported' it to the rider's center of mass, that is one of the basic rules of vector calculus.

In this description, the proper relationship is:

$$\mathbf{F = M\,V^2 / R}$$

A numerical example shows us that if the mass of the rider is 100 kg (bicycle included, "**g**" = 10 $m.s^{-2}$), the speed 10 meters per second, and the radius of its path 100 meters, the intensity of the guiding force is 100 newtons. Furthermore, an accurate measurement of the cyclist's leaning angle relative to the vertical (from a photograph, for example) show us that, in these conditions, this angle is exactly equal to 5.7 degrees.

- Drawing 2: this is the imaginary description, called '*static*'. The rotating base converts to fixed base. Concrete translation: the cyclist is motionless, he is no longer moving. But it leans to one side! To prevent fall, the equilibrium requires the presence of a force **F'**, weel known as '*centrifugal force*' (blue arrow in the drawing 2).

This force is supposed to act on the cyclist's center of mass (disregarding the mass of the bicycle). But does this famous force really exist? No, because nobody has ever seen a cyclist, both motionless and able to stay balanced in such a position.

This force is called centrifugal, that means '*a force that moves away from the center*'. Is this term appropriate? Because the cyclist is motionless, there is neither trajectory, nor radius nor center. The correct name of this force is 'fictive force', 'imaginary force' or 'pseudo-force'.

Another common mistake: the relationships between quantities. Indeed, contrary to popular belief, the famous magic formula **F = MV² / R** can never applies to this description because the speed is always zero. An attempt to numerical application confirms it:

$$\forall\,\mathbf{M},\ \forall\,\mathbf{R} \neq \mathbf{0},\ \text{pour}\ \mathbf{V} = \mathbf{0},\ \mathbf{F} = \mathbf{MV^2/R} = \mathbf{0/R} = \mathbf{0}\ !...$$

Here is this famous mathematical proof! And that's not enough! What value to give to R? How measuring the trajectory radius of a motionless object? It is clear, clean and definitive: this formula does not work, it leads to a dead end!

So, how to calculate the intensity of centrifugal force? In truth, the only relationship that fits this description is the following:

$$\mathbf{F'} = -\mathbf{P} \, . \, \textbf{tangent } \alpha$$

In this relation, P is the weight of the mass and α the leaning angle relative to the vertical. Here, the [**–**] sign is decisive: it specifies the spatial orientation of this force that, obviously, should act on the opposite side of the tilt. We will see later why that sign is so important.

The digital implementation of this equation shows us that retaining a motionless cyclist with a mass of 100 kilograms (bicycle understood, "**g**" = 10 $m.s^{-2}$) in a tilted position of 5.7 degrees from the vertical, needs a force of 100 newtons.

A 100 newton force? What a curious coincidence! Or rather: what an extraordinary values correspondence, what marvelous precision of the physics, for chance and coincidences obviously do not have any place here!

These calculations show that, for equal leaning angle, the guiding force (dynamic description) and the centrifugal force (static description) have the same intensity! Hence the confusion.

Beware! The accuracy of the calculation and the equality of the result do not prove the reality of the concept. Only one of these two forces is real, that is the guiding force. The other is purely imaginary, it is the centrifugal force.

And now, we will demonstrate that these two forces, and therefore the two descriptions which they relate, can neither complement nor mix!

- Drawing 3: it is a mixture of the two previous descriptions, a sort of fusion against nature intended to bring clear and definitive evidence that these two descriptions are totally incompatible.

In this description, the two forces coexist, but the vector calculation shows that these two forces of equal intensity but of opposite directions, cancel each other:

$$\mathbf{F} + \mathbf{F'} = \mathbf{100 - 100} = \mathbf{0} \;!$$

How to interpret this result? A vector sum of zero means that everything happens as if these two forces were not acting: the cyclist is now completely released!

That is, if we reason in dynamics, the cyclist (who is in motion) remains tilted but moves a straight path, he cannot describe any circular trajectory! And if we reason in statics, the cyclist (who is motionless) immediately falls, he cannot stay motionless and leaning at the same time!

This reasoning by contradiction must convince us that it is imperative to choose one of these two descriptions and forget the other! QED!

That is an absolute rule: never mix a rotating base and a fixed base, a dynamic description and a static description, the present and the conditional, reality and fiction!

Static or dynamic: the true definition!

The above brings us to these two definitions:

'Is called a perfectly static description in which a moving base is considered motionless.'

'*We call dynamic a description of the real movement and its causes*.'

Caution! Physics abhors mix of genres. We insist again on the formal and absolute prohibition to merge the two descriptions. Please note a common mistake: confusion of the static and dynamic descriptions (see ADILCA files "*centrifugal force*" and "*inertial force*").

What misunderstandings, as setbacks, blunders for those, students or teachers who do not respect these precautions!

The principle of reciprocity

The distinction between statics and dynamics implies a significant consequence concerning the application of Newton's third principle, or principle of reciprocity, a principle often forgotten or misunderstood.

What about this famous principle?

'*Any mass on which a force is acting, reacts by a reciprocal action of equal intensity, but opposite direction.*'

How does this principle apply in statics?

Answer: it does not apply because a static description is an imaginary description, there is no interaction.

How does this principle apply in dynamics?

Answer: As we have just seen, the guiding force is a horizontal force which exerts on the mass of the cyclist (we neglect the mass of the bicycle) by means of the tires of the machine in contact with the ground.

We deduce that the mass of the cyclist exerts a horizontal thrust on the globe by means of the bicycle tires, of equal intensity to that of the guiding force, but in the opposite direction[**].

The rotational movement of the Earth is not disturbed by this phenomenon, given the ratio of the masses[***].

Centrifugal force: the true definition!

The distinction between statics and dynamics brings us to these two original and unpublished definitions of centrifugal force:

'We call centrifugal force a transverse force that should be acting on the center of mass of a motionless bicycle (disregarding the mass of the bicycle) to keep it in balance with the same leaning angle as that can be observed when describing a circular trajectory. This definition applies only in statics and is void if the bicycle is in motion.'

For a car, this definition becomes:

'We call centrifugal force a transverse force that should be acting on the center of mass of a motionless car to create on tires and suspension a comparable effect to that which can be observed when the car is in motion and describes a circular trajectory. This definition, only valid in statics, lapses if the car is moving.'

A purely imaginary force!

Emphasize the first requirement of this definition: the vehicle must be motionless. Then underline the hypothetical nature of this force, made clear by the use of the conditional '*force should be exercised*'... and finally, the technical impossibility to exercise directly any force to the center of mass.

Moreover, no one has ever seen such a force appearing spontaneously. We therefore deduce that the centrifugal force is an imaginary force. This is an apparent force, a fictive or pseudo-force, i.e. a force that has no real existence.

The only force that really exists is exercised in contact with the ground, is the guiding force. It is the only force that deflects the straight path of land vehicles, and allows circular trajectories. There is no other force into play on this occasion!

Proof by the water bottle

How to distinguish a static description of a dynamic description?

The experiment called 'water bottle' can give an answer! It consists to partially fill a plastic water bottle, preferably with syrup (mint, grenadine...) to color the content and then install the bottle flat across the handlebars or tank of a motorcycle.

The experiment called 'water bottle' can reveal the true nature of the description. In dynamics, the liquid surface remains perpendicular to the motorcycle's plane of symmetry. In statics, the liquid falls on the side where the machine leans.

Then just go for a ride and observe the movements of the liquid during variations in trajectories: except brutal driving or bike's shaking, the surface remains perpendicular to the axis of symmetry of the motorcycle. In statics, the liquid falls on the leaning side.

This experience is very useful: when a teacher draws some forces, ask him how will behave a liquid inside a bottle... You will know right away if the teacher reasons in statics, in dynamics, or if he is wrong!

What is the use of an imaginary description?

Beyond the controversy as fun as sterile, the real question to ask is this: why did physicists need to invent imaginary descriptions? Reality is not enough?

Incidentally, this exercise of intellectual acrobatics, used by many teachers, constitutes a kind of 'donkeys bridge' intended to sort out the pseudo-scientific (those who confuse the pseudo-forces with the real forces!).

But the real explanation is not there. To the physicist, this intellectual construction is justified in the consistency of all descriptions, even theoretical and fictional.

Thus Physics abhors a vacuum and physicist projects its rigorous logic everywhere, including in the abstract!

Fictive forces are concepts that serve only to satisfy the need for coherence of the theory. Nevertheless, these forces are purely imaginary, they have no real existence. To claim otherwise is fraud or incompetence.

Conclusion

Imposture is not the concept of centrifugal force!

One of intellectual imposture is the confusion between dynamics and statics, between reality and fiction, between present and conditional.

To avoid any misunderstanding, the least precaution should be to clearly state the nature of the description and add that, in the case of a static description, it can only be an imaginary description.

It is obvious: any mixture of static and dynamics is prohibited.

One consequence of this distinction is that the statics involve fictive forces, also called apparent forces, imaginary forces or pseudo-forces, among which centrifugal force. Because, the other intellectual imposture is to believe or make believe that the centrifugal force really exists!

The other consequence is that the principle of reciprocity do not apply in statics, it can only apply in dynamics.

Last but not least, the famous magic formula $F = M.V^2 / R$ used almost everywhere can be applied only to a dynamic description, it can therefore express only one force: the guiding force.

(*) *Principle of inertia of Isaac Newton:* 'A mass on which no force acts, describes a perfectly rectilinear trajectory.' *The concept of force derives from this principle:* 'A force refers to any cause able to deflect the trajectory of a mass.'

(**) *The cyclist is perfectly aware of this reciprocal action, which gives him the feeling of weighing more heavily on the tires of the bicycle.*

(***) *Mass of the cyclist and bicycle: 100 kg; Earth mass: 6×10^{24} kg, a ratio of 1 for 6×10^{22} (which means a number equal to 6 followed by 22 zeros). The guiding force and its interaction are equal, but their effects depend on the masses on which they are acting. It is a concrete application of the second principle of the dynamics enunciated by Isaac Newton and which results in this equation: [**Y = F / M**]. By examining the limits of this function, we see that if mass tends, if not towards infinite, at least towards a very large number, the acceleration tends, if not towards zero, at least towards a very small number. This explains why the globe remains insensitive to the cyclist's circular trajectory.*

III. CENTRIFUGAL FORCE AND GUIDING FORCE: THE CALCULATION MODE

1. Guiding force

$$F = M \cdot V^2 / R$$

F: guiding force, expressed in **N**
M: mass, expressed in **kg**
V: speed, expressed in $\mathbf{m.s^{-1}}$
R: radius of trajectory, expressed in **m**
consistency of the units: $\mathbf{F} = kg \cdot (m.s^{-1})^2 \cdot m^{-1} = kg \cdot (m^2.s^{-2} \cdot m^{-1}) = kg.m.s^{-2} = \mathbf{N}$

Example: calculate the guiding force acting in contact with the ground and maintaining a bicycle + cyclist set of 100 kilograms (220 lb) on a circular trajectory of 100 meters radius (330 ft) at a speed of 10 meters per second (22.4 mph):

$$\mathbf{F} = 100 \times 10^2 / 100 = 100 \times 100 / 100 = 100 \ \mathbf{N}$$

2. Lean angle

$$\text{Tangent } \alpha = F / P$$

α: lean angle, dimensionless;
F: guiding force, expressed in **N**
P: weight, expressed in **N**
consistency of the units: N / N = dimensionless

Example: calculate the lean angle when a 1,000 N bicycle + cyclist set describes a circular trajectory with a guiding force of 100 N:

$$\textbf{Tangent } \boldsymbol{\alpha} = 100 / 1{,}000 = 0.1$$

$$\alpha = 5.7°$$

3. Centrifugal force

$$F' = -P \cdot \text{tangent } \alpha$$

F': centrifugal force, expressed in **N**
P: weight, expressed in **N**
α: lean angle, dimensionless.
consistency of the units: $\mathbf{F'} = \mathbf{kg.m.s^{-2}}$

Example: calculate the centrifugal force that should be acting on the center of mass of a motionless bicycle + cyclist set weighing 1,000 N, in order to maintain it in equilibrium despite a 5.7 degrees lean angle:

F’ = – 1,000 x tangent 5.7° = – 1,000 x 0.1 = – 100 **N**

Note 1: this force is commonly referred to as "*centrifugal force*" which is an incorrect name since there is neither trajectory nor center (the bicycle + cyclist set remains motionless all the time). The scientific name of this force is: imaginary force, fictional force, pseudo-force, or inertial force.

Note 2: the sign [–] is required, it specifies that the spatial orientation of this force conflicts the logic of the movement.

Note 3: beware to misinterpretation, the numerical equality of results does not allow the interchangeability of descriptions, concepts or reasoning.

Note 4: the different calculations must be done in the order indicated. It is indeed impossible to directly calculate the centrifugal force without performing the intermediate calculations detailed above.

Note 5: Every scientific approach goes through four steps:

- form observation to *experiment* (here: a bicycle which describes a circular path);
- from experiment to *measurements* (here: the mass of the set, its speed and the radius of its trajectory);
- from measurements to *calculations* (here: calculation a guiding force and a lean angle);
- from calculations to *reasoning* (here: the concept of centrifugal force).

This transition from concrete to abstract reasoning, from the real to the imaginary, has often been short-circuited, hence the confusion or misunderstanding about centrifugal force.

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